Exploring the Distribution of Local Neighborhood Structures in Large Networks

Shenghui Cheng\(^{(2)}\), Claudia Dahl\(^{(1)}\), Joachim Giesen\(^{(1)}\), Philipp Lucas\(^{(1)}\) and Klaus Mueller\(^{(2)}\)

1: Friedrich-Schiller-Universität Jena, Germany
2: SUNY Stony Brook, USA and SUNY, Korea
Take aways / Outline
Take aways / Outline

1. Idea: node feature based on the geometry of the local neighborhood of a node
   
   -> *feature extraction pipeline*
Take aways / Outline

1. Idea: node feature based on the geometry of the local neighborhood of a node
   -> feature extraction pipeline

2. It is useful! 😊
   -> demo!
Exploratory network analysis

Exploration

Outlier
Motif
Cluster
Central Node
Exploratory network analysis

Feature Extraction

\[
\begin{pmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3 \\
\lambda_4 \\
\vdots
\end{pmatrix}
\]

Degree
Betweenness
Closeness

Exploration

Outlier
Motif
Cluster
Central Node
One feature vector per node
One feature vector per node
One feature *vector* per node
One feature *vector* per node
What kind of features? Based on local neighborhoods!
What kind of features? Based on local neighborhoods!

Node
What kind of features?  Based on local neighborhoods!

Neighbors with respect to some distance measure
What kind of features? Based on local neighborhoods!

Node

Local Neighborhood

Characterization of Neighborhood

Neighbors with respect to some distance measure
What kind of neighborhood?  Based on geometric properties!
What kind of neighborhood? Based on geometric properties!

Graph
What kind of neighborhood? Based on geometric properties!
What kind of neighborhood? Based on geometric properties!

Graph → Spectral embedding → Point Cloud → k-nearest neighbors → Local Neighborhood (per node)
What kind of characterization? Based on geometric properties
What kind of characterization?  Based on geometric properties
What kind of characterization? Based on geometric properties

Local Neighborhood (per node)

Principal Components of a Local Neighborhood

principal component analysis (PCA)
What kind of characterization? Based on geometric properties

Local Neighborhood (per node)

Principal Components of a Local Neighborhood

Characterization of a Local Neighborhood

principal component analysis (PCA)

length and relative angles

$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \omega_1 \\ \omega_2 \end{pmatrix}$
Shape of a Neighborhood

Local Neighborhood (per node)
Shape of a Neighborhood

Local Neighborhood (per node)

Principal component analysis (PCA)

Ellipsoidal

*Local Neighborhood* (per node)

Principal component analysis (PCA)

Ellipsoidal
Shape of a Neighborhood

Local Neighborhood (per node)

Principal component analysis (PCA)

Ellipsoidal

Spherical
Graph → Spectral embedding → Point Cloud
Graph

Spectral embedding

Point Cloud

k-nearest neighbors

Local Neighborhood (per node)
Graph

Spectral embedding

Point Cloud

k-nearest neighbors

Local Neighborhood (per node)

Principal Components of Neighborhood

Local PCA
Graph

Spectral embedding

Point Cloud

k-nearest neighbors

Local Neighborhood (per node)

Shape of Neighborhood

local PCA

Feature Vectors

Principal Components of Neighborhood

\[
\begin{align*}
\begin{pmatrix}
\mu_{1,1} \\
\mu_{1,2} \\
\mu_{1,3} \\
\vdots \\
\omega_{1,1} \\
\omega_{1,2} \\
\omega_{1,3} \\
\vdots
\end{pmatrix},
\begin{pmatrix}
\mu_{2,1} \\
\mu_{2,2} \\
\mu_{2,3} \\
\vdots \\
\omega_{2,1} \\
\omega_{2,2} \\
\omega_{2,3} \\
\vdots
\end{pmatrix},
\begin{pmatrix}
\mu_{3,1} \\
\mu_{3,2} \\
\mu_{3,3} \\
\vdots \\
\omega_{3,1} \\
\omega_{3,2} \\
\omega_{3,3} \\
\vdots
\end{pmatrix},
\end{align*}
\]
Demo!